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Extensions of $AdS_5 \times S^5$ and the plane-wave superalgebras and their realization in the tiny graviton matrix theory

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Abstract: In this paper we consider all consistent extensions of the $AdS_5 \times S^5$ superalgebra, $psu(2,2|4)$, to incorporate brane charges by introducing both bosonic and fermionic (non)central extensions. We study the Inönü-Wigner contraction of the extended $psu(2,2|4)$ under the Penrose limit to obtain the most general consistent extension of the plane-wave superalgebra and compare these extensions with the possible BPS (flat or spherical) brane configurations in the plane-wave background. We give an explicit realization of some of these extensions in terms of the Tiny Graviton Matrix Theory (TGMT) [14] which is the $0+1$ dimensional gauge theory conjectured to describe the DLCQ of strings on the $AdS_5 \times S^5$ and/or the plane-wave background.

Keywords: Extended Supersymmetry, M(atrix) Theories, Penrose limit and pp-wave background.

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1. Introduction

Historically supersymmetry (SUSY) algebras arose as extensions of Poincaré algebra of a D-dimensional space-time by fermionic generators, the supercharges. The first extension to the supersymmetry algebra constructed in this way, the super-Poincaré algebra, appeared when the number of supercharges were taken to be a multiple (usually taken to be a power of two) of dimension of the smallest spinor representation of the corresponding Poincaré algebra. Such extensions usually come under the title of $\mathcal{N}$-extended SUSY algebras [1]. In this extended versions of superPoincaré the bosonic part of the superalgebra is extended by an "internal" R-symmetry group, which in four dimensions is a $\mathrm{U}(\mathcal{N})$ symmetry, under which the supercharges are in fundamental representations. It is then natural to ask for other possible extensions of the $\mathcal{N}$-extended algebras. If such an extension exists, then we end up with an extended SUSY algebra which contains the original algebra as a subalgebra. Generically speaking these extensions can appear in the bosonic or fermionic sectors of the SUSY algebra and they can be central or non-central. Central extensions are those which are at the center of the corresponding Poincaré algebra and hence they are necessarily

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scalars in space-time. There are also p-form extensions, which are all non-central (for example see [2] and references therein). These p-form extensions, however, generically commute with the momenta and the super(Poincaré)-charges and hence central in this sense.[1] As is well-known presence of these p-form extensions are necessitated by dualities of string theories, as they correspond to the charges of extended objects, p-branes, e.g. see [2 − 4].

For the *non-central* extensions of a given superalgebra one should check the closure and the Jacobi identities for the extended algebra. Generically, as first noted in [5], closing the algebra amounts to adding fermionic as well as bosonic p-forms, i.e. fermionic brane charges. Existence of the super-p-form charges are necessary for construction of supersymmetric Wess-Zumino terms in p-brane actions [5].

It is not always necessary to build a SUSY algebra on a Poincaré or Lorentz algebra and one can construct supersymmetric versions of all classical Lie algebras by adding fermionic generators in the spinor representations of these algebras and demanding their closure. Superalgebras obtained in this way all fall into Kac-Nahm classification of superalgebras [6, 7]. These algebras become of great importance in string theory once we consider theories on spaces which are not asymptotically flat, but still supersymmetric.

Regardless of the dimension of space-time in which the superalgebra is defined the maximal number of supercharges in a physically viable theory, which does not contain higher than spin two fundamental particles, cannot exceed 32. In this paper hence our focus would be on the maximal superalgebras which are not based on superPoincaré. The most famous of examples of such cases are the maximally supersymmetric $AdS_p \times S^q$ $(p, q) = (5, 5)$, $(4, 7)$ and $(7, 4)$ geometries and the corresponding plane-waves in ten or eleven dimensions [8]. For the $AdS_5 \times S^5$ the relevant algebra is $psu(2, 2|4)$ which will be reviewed and explicitly presented in section 2.1. As has been discussed in [9 − 11] to find the maximally possible extension of this algebra one should add other fermionic, as well as p-form generators. Although pure $psu(2, 2|4)$ is a subsuperalgebra of $ops(1|32)$, it is not the case for its maximal extension. While the bosonic part of the maximally extended PSU$(2, 2|4)$ is a subgroup of bosonic part of OSp$(1|32)$, the whole superalgebra is a contraction of $osp(1|32)$. This algebra will be presented and discussed in some detail in 2.2. Interestingly and intriguingly $osp(1|32)$ also appears as the maximal possible extension of all the other known flat space maximal superalgerbas, namely ten dimensional $\mathcal{N} = 2$ superPoincaré [3, 4], eleven dimensional $\mathcal{N} = 1$ superPoincaré [5, 12].

In this paper we will study the maximal extensions of the AdS superalgebras and the behavior under the Penrose limit. In this way we will obtain the most general extension of the other class of maximally supersymmetric non-flat supergravity backgrounds, namely the plane-wave geometries. This is done in section 3. In this paper we will discuss the $psu(2, 2|4)$ in detail. The computations for the $osp(8^*|4)$ and $osp(4|8)$ cases can be performed in a similar way. The extended eleven dimensional plane-wave superalgebra has been discussed in [13].

[1] As discussed e.g. in [2], an extension of the SUSY algebra which is not central, upon dimensional reduction, may appear as a central extension in the lower dimensional superalgebra.

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The extension of the plane-wave superalgebras are of particular interest because for these cases we have matrix theory formulations for the DLCQ of type IIB strings and/or the M-theory on these backgrounds [14, 15]. As discussed in [14] both of these matrix theories are "tiny graviton matrix theories", the tiny three brane and membrane theories, respectively. Within these matrix models one may then find explicit representations of these algebras and their super-extensions in terms of matrices. These are the charges corresponding to BPS objects in these matrix models, and hence the extended brane-type objects in the corresponding string or M-theories. In section 4, we will focus on the representation of the extensions of the ten dimensional plane-wave superalgebra in the tiny graviton matrix theory (TGMT) and their brane interpretation.

In the last section we conclude by summary of our results, outlook and remarks. In the two appendices we have gathered a review of the relevant fermionic notations employed in the paper and a brief review on the tiny graviton matrix theory.

2. Extensions of the $psu(2,2|4)$ superalgebra

In this section we review the $psu(2,2|4)$ algebra and its possible extensions. We employ the fermionic notations developed in [16]. To be self-contained, we have explained our fermionic notations in the appendix A.

Note on the classification of superalgebras. According to the Nahm classification of superalgebras [6], generically in any superalgebra, besides the central generators, two kind of bosonic groups are involved. For example in the four dimensional $\mathcal{N}$-extended SUSY algebras, the four dimensional Poincaré $ISO(3,1)$ and the U($\mathcal{N}$) R-symmetry group. The supercharges are then in the spinor representation of both of these groups. (The U(1) part of the U($\mathcal{N}$) R-symmetry is generically anomalous in the supersymmetric gauge theories). This can be generalized by taking the supercharges to be in spinor representation of any two classical Lie groups. The most famous cases are $su(m|n)$ algebras in which the $2mn$ (real) supercharges are in the spinor representation of $su(m)$ and $su(n)$. That is, we can denote the supercharges by $Q_{a\alpha}$, where a, α, $a = 1, 2, \dots, m$, $\alpha = 1, 2, \dots, n$ are the fundamental indices of SU(m) and SU(n). The bosonic part of this superalgebra, besides $su(m) \times su(n)$, generically contains an extra U(1). For the special case of $m = n$, however, this U(1) factor becomes central to the whole algebra and maybe extracted out. In which case to emphasize absence of the U(1), the algebra is denoted by $psu(m|m)$. The examples of this algebras are $psu(2,2|4)$ whose bosonic part is $su(2,2) \simeq so(4,2)$ times $su(4) \simeq so(6)$, and $su(4|2)$ with the bosonic part $su(4) \times su(2) \times u(1) \simeq so(6) \times so(3) \times u(1)$ [17]. The former is a superalgebra with $2 \times 4 \times 4 = 32$ supercharges and the latter with $2 \times 4 \times 2 = 16$.

The next supergroups/algebras relevant to the 11 dimensional AdS cases are OSp($m|n$). The bosonic part of which are $so(m)$ (or more precisely Spin(m)) times $USp(n)$. (In our conventions $USp(2n) \simeq \mathrm{Spin}(2n+1)$.) The examples of these algebras appearing in the M-theory backgrounds are $osp(8|4)$ with the bosonic part $so(8) \times usp(4) \simeq so(8) \times so(3,2)$, $osp(8^*|4)$ whose bosonic part is $so(8^*) \times usp(4) \simeq so(6,2) \times so(5)$ and finally $osp(1|32)$ with the bosonic part $usp(32)$. All these superalgebras have 32 supercharges. Further discussions on these superalgebras may be found in [18].

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2.1 Unextended $psu(2,2|4)$ superalgebra

PSU$(2,2|4)$ is the supergroup of the $\mathcal{N}=4$ four dimensional Yang-Mills superconformal field theory. As explained above, in the most natural notations, the supercharges of this algebra carry two four dimensional Weyl indices of $so(6)$ and $so(4,2)$. That is, the supercharges are labeled as $Q_{I\hat{J}}$, where I and $\hat{J}=1,2,3,4$ are respectively the Weyl indices of $so(6)$ and $so(4,2)$. This is the notations and conventions used in [16].[2] The superalgebra in this notation takes the compact form of [16]

$$[\mathbb{J}_{AB},\mathbb{J}_{CD}] = i(\delta_{A[C}\mathbb{J}_{B]D} - \delta_{B[C}\mathbb{J}_{A]D})\ , \quad [\mathbb{J}_{\hat{\mu}\hat{\nu}},\mathbb{J}_{\hat{\rho}\hat{\lambda}}] = i(\eta_{\hat{\mu}[\hat{\rho}}\mathbb{J}_{\hat{\nu}]\hat{\lambda}} - \eta_{\hat{\nu}[\hat{\rho}}\mathbb{J}_{\hat{\mu}]\hat{\lambda}}) \tag{2.1a}$$

$$\{\mathcal{Q}_{I\hat{J}},\mathcal{Q}^{\dagger K\hat{L}}\} = 2(i\gamma^{\hat{\mu}\hat{\nu}})_{\hat{I}}{}^{\hat{K}}\mathbb{J}_{\hat{\mu}\hat{\nu}}\ \delta_J{}^L + 2(i\gamma^{AB})_J{}^L\mathbb{J}_{AB}\ \delta_{\hat{I}}{}^{\hat{K}}\ , \quad \{\mathcal{Q}_{I\hat{J}},\mathcal{Q}_{K\hat{L}}\} = 0 \tag{2.1b}$$

$$[\mathbb{J}_{AB},\mathcal{Q}_{I\hat{J}}] = (i\gamma_{AB})_I{}^K\mathcal{Q}_{K\hat{J}} \quad , \quad [\mathbb{J}_{\hat{\mu}\hat{\nu}},\mathcal{Q}_{I\hat{J}}] = (i\gamma_{\hat{\mu}\hat{\nu}})_{\hat{J}}{}^{\hat{L}}\mathcal{Q}_{I\hat{L}} \tag{2.1c}$$

where $A,B=1,2,\ldots,6$ are the fundamental SO(6) indices and $\mathbb{J}_{AB}$ are generators of $so(6)$. The hatted Greek indices run over $-1,0,\ldots,4$ and are fundamental SO$(4,2)$ indices, whose algebra generators are denoted by $\mathbb{J}_{\hat{\mu}\hat{\nu}}$ and $i\gamma_{AB}$, $i\gamma_{\hat{\mu}\hat{\nu}}$ are the commutators of the SO(6) and SO$(4,2)$ γ-matrices in the Weyl representation and hence they are 4×4 hermitian matrices. For more details see appendix A.

2.2 The extended $psu(2,2|4)$ superalgebra

In this subsection we study the most general superalgebra permitted by just group theory considerations, we add additional objects (terms) which fall in different allowed representations of the bosonic part of the superalgebra. Later we give physical interpretations for this extra terms using field theory description, and also realizations in the langauge of the corresponding matrix theory. The physical significance of these extensions lies in the fact that they correspond to charges of different extended objects. Explicitly, in order to have supersymmetric (BPS) objects in the theory, we have to add their charges into the superalgebra.

According to our conventions, as discussed in appendix A, spinorial supercharges carry two indices corresponding to fundamentals of $su(2,2), su(4)$ as $\mathcal{Q}_{I\hat{J}}$ which is the direct product of two representations $\mathbf{4}\otimes\hat{\mathbf{4}}$. When we anticommute $\mathcal{Q},\mathcal{Q}^\dagger$, each fundamental properly multiplies with its antifundamental counterpart to give

$$\mathbf{4}\otimes\bar{\mathbf{4}} = \mathbf{1}\oplus\mathbf{15}$$

Therefore, group theory dictates that the allowed extensions which can appear in the right-hand-side of the $\mathcal{Q},\mathcal{Q}^\dagger$ anticommutator should fall into the following representations of SO$(4,2)\times S(6)$:

$$(\mathbf{1}\oplus\mathbf{15})\otimes(\mathbf{1}\oplus\mathbf{15}) = (\mathbf{1},\mathbf{1})\oplus(\mathbf{1},\mathbf{15})\oplus(\mathbf{15},\mathbf{1})\oplus(\mathbf{15},\mathbf{15}) \tag{2.2}$$

[2] Note, however that, although the most convenient one, this is not the notation usually used for labeling the supercharges of this algebra. Usually the $Q_{I\hat{J}}$ supercharges are decomposed in terms of $so(3,1)$ spinors, and as superPoincaré and super conformal charges S, see e.g. [19]. The latter labeling seems more suitable from the $\mathcal{N}=4$ super-Yang-Mills theory.

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Similarly the extensions to $\mathcal{Q}, \mathcal{Q}$ anticommutator, as

$$\mathbf{4} \otimes \mathbf{4} = \mathbf{6}_a \oplus \mathbf{10}_s,$$

must fall into

$$\mathrm{Sym}[(\mathbf{6}_a \oplus \mathbf{10}_s) \otimes (\mathbf{6}_a \oplus \mathbf{10}_s)] = (\mathbf{6}_a, \mathbf{6}_a) \oplus (\mathbf{10}_s, \mathbf{10}_s) \tag{2.3}$$

of $\mathrm{SO}(4,2) \times S(6)$. Note that $(\mathbf{6}_a, \mathbf{10}_s) \oplus (\mathbf{10}_s, \mathbf{6}_a)$ being antisymmetric cannot appear in the $\mathcal{Q}, \mathcal{Q}$ anticommutator.

Therefore, the most general extended anticommutators read as

$$\begin{aligned} \{\mathcal{Q}_{I\hat{J}}, \mathcal{Q}^{\dagger K\hat{L}}\} =& \chi\ \delta_I{}^K\ \delta_{\hat{J}}{}^{\hat{L}} \\ &+ \mathbb{J}_{AB}(i\gamma^{AB})_I{}^K\ \delta_{\hat{J}}{}^{\hat{L}} + \delta_I{}^K\ \mathbb{J}_{\hat{\mu}\hat{\nu}}(i\gamma^{\hat{\mu}\hat{\nu}})_{\hat{J}}{}^{\hat{L}} \\ &+ \mathcal{R}_{AB\hat{\mu}\hat{\nu}}\ (i\gamma^{AB})_I{}^K\ (i\gamma^{\hat{\mu}\hat{\nu}})_{\hat{J}}{}^{\hat{L}} \end{aligned} \tag{2.4}$$

$$\begin{aligned} \{\mathcal{Q}_{I\hat{J}}, \mathcal{Q}_{K\hat{L}}\} =& \mathcal{Z}_{A\hat{\mu}}\ (\gamma^A)_{IK}\ (\gamma^{\hat{\mu}})_{\hat{J}\hat{L}} \\ &+ \mathcal{Z}^{++}_{ABC\hat{\mu}\hat{\nu}\hat{\rho}}\ (\gamma^{ABC})_{\{IK\}}\ (\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}})_{\{\hat{J}\hat{L}\}} \end{aligned} \tag{2.5}$$

Note that γ^A_{IK} and $\gamma^{\hat{\mu}}_{\hat{J}\hat{L}}$ are antisymmetric in IK and $\hat{J}\hat{L}$ indices while γ^{ABC} and $\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}}$ are symmetric. $\mathcal{Z}^{++}_{ABC\hat{\mu}\hat{\nu}\hat{\rho}}$ is self-dual in both SO(6) and SO(4, 2) indices, that is

$$\frac{1}{3!}\epsilon^{ABC}{}_{DEF}\ \mathcal{Z}^{++}_{ABC\hat{\mu}\hat{\nu}\hat{\rho}} = +\mathcal{Z}^{++}_{DEF\hat{\mu}\hat{\nu}\hat{\rho}}\ , \quad \frac{1}{3!}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}}{}_{\hat{\alpha}\hat{\beta}\hat{\lambda}}\ \mathcal{Z}^{++}_{ABC\hat{\mu}\hat{\nu}\hat{\rho}} = +\mathcal{Z}^{++}_{DEF\hat{\alpha}\hat{\beta}\hat{\lambda}}\ . \tag{2.6}$$

Since 6_a and 10_a representations of SU(4) and SU(2, 2) are complex valued the corresponding extensions, $\mathcal{Z}_{A\hat{\mu}}$ and $\mathcal{Z}^{++}_{ABC\hat{\mu}\hat{\nu}\hat{\rho}}$ are also complex while χ and $\mathcal{R}_{AB\hat{\mu}\hat{\nu}}$, as well as $\mathbb{J}_{\hat{\mu}\hat{\nu}}$ and $\mathbb{J}_{AB}$ are hermitian.

The necessary and sufficient condition for the consistency of the extended algebra, is that it satisfies different Jacobi identities among fermionic/bosonic generators. The extensions, except for the χ, carry SO(6) and SO(4, 2) tensor indices and hence have non-trivial commutators with $\mathbb{J}_{\hat{\mu}\hat{\nu}}$ and $\mathbb{J}_{AB}$ generators i.e. they are *not* central to the original $psu(2,2|4)$ superalgebra. Therefore, closure of the algebra and the Jacobi identities forces non-zero commutators between the supercharges Q and the extensions $\mathcal{Z}$ and $\mathcal{R}$. Moreover, in order to close the algebra we should also add more fermionic generators, i.e. fermionic counterparts to the extensions or the fermionic brane charges. This phenomenon was first noted by Sezgin in the context of the M-theory superalgebra [5] and then extended to the other cases. For more details and the structure of the new (anti)commutators see [10 – 13].

Physically, $\chi, \mathcal{R}_{AB\hat{\mu}\hat{\nu}}, \mathcal{Z}_{A\hat{\mu}}, \mathcal{Z}^{++}_{ABC\hat{\mu}\hat{\nu}\hat{\rho}}$ respectively correspond to charges of D-instanton, D3, F1/D1, NS5/D5 branes. From the effective field theory description and its Wess-Zumino term, it can be shown that they are spatial integral of total derivatives, and are topological rather than Noether charges [9]. In fact in the above extensions, the *spatial* components of the extensions corresponds to brane *dipole* moment charges. For example, as we will see explicitly in section 4 from the tiny graviton Matrix theory construction $\mathcal{R}_{AB\hat{\mu}\hat{\nu}}$ leads to the RR dipole moment of spherical (giant graviton) D3-branes.

One may consider extensions of the eleven dimensional AdS superalgebras, $osp(8|4,\mathbb{R})$ and $osp(8^*|4)$. It has been argued that the maximal extension of both of these lead to

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a contraction of $osp(1|32)$ [9, 12, 13]. Intriguingly, a different contraction of the same $osp(1|32)$ algebra appears as the maximal extension of $psu(2,2|4)$ we discussed above. (In our above discussions we did not add the other needed fermionic generators to make the $osp(1|32)$ structure explicit. More discussions on this maybe found in [10, 11].) It seems that there is a uniqueness theorem [9, 12], that the maximal extension of any superalgebra with 32 supercharges is either $osp(1|32)$ or a contraction of that.

3. Inönü-Wigner contraction of the extended $psu(2,2|4)$

Parallel to the Penrose limit which, at the level of the geometry, takes us from the $AdS_5 \times S^5$ solution to the plane-wave solution, there is a complementary process, known as Inönü-Wigner contraction [20], which does the same at the level of the (super)algebra [21]. In this section, following notations of [16], we first briefly review the action of the Penrose limit over the pure (unextended) super-isometry group of the $AdS_5 \times S^5$ which exactly produces super-isometry group of the plane-wave and then contract the extended super-isometry group of the $AdS_5 \times S^5$ with all the possible extensions to obtain the most general extensions of the plane-wave superalgebra. In this way we can trace back the extensions of the plane-wave algebra to their counterparts in the AdS and hence read their physical interpretations.

Contraction of the bosonic part. The bosonic part of isometries is $\mathrm{SO}(4,2) \times \mathrm{SO}(6)$ with the generators $\mathbb{J}_{\hat{\mu}\hat{\nu}}$ and $\mathbb{J}_{\hat{A}\hat{B}}$. In order to contract the algebra, it is more convenient to decompose them as

$$\mathbb{J}_{\hat{\mu}\hat{\nu}} = \left\{ J_{ij},\ L_i = \frac{1}{R}(J_{-1i} + iJ_{0i}),\ K_i = \frac{-1}{R}(J_{-1i} - iJ_{0i}),\ \mu R^2 P^+ + \frac{1}{2\mu}\mathbf{H} = J_{-10} \right\} \tag{3.1a}$$

$$\mathbb{J}_{\hat{A}\hat{B}} = \left\{ J_{ab},\ L_a = \frac{1}{R}(J_{5a} + iJ_{6a}),\ K_a = \frac{-1}{R}(J_{5a} - iJ_{6a}),\ \mu R^2 P^+ - \frac{1}{2\mu}\mathbf{H} = J_{56} \right\}, \tag{3.1b}$$

where $i, a = 1,2,3,4$. In the above parametrization the Penrose limit is $R \to \infty$ while keeping $J_{ij}, J_{ab}, L_i, K_i, L_a, K_a, P^+$ and $\mathbf{H}$ fixed.

Contraction of the fermionic part. In order to perform the contraction on the fermionic part it is convenient to adopt the $\mathrm{SO}(4) \times \mathrm{SO}(4)$ notation for the fermions and scale them as follows (see [16] or the appendix A.2),

$$\mathcal{Q}_{I\hat{J}} \to \left(\sqrt{\mu}R\ q_{\alpha\beta}, \sqrt{\mu}R\ q_{\dot{\alpha}\dot{\beta}}, \frac{1}{\sqrt{\mu}}Q_{\alpha\dot{\beta}}, \frac{1}{\sqrt{\mu}}\ Q_{\dot{\alpha}\beta} \right), \tag{3.2}$$

send $R \to \infty$ while keeping the q and Q in the right-hand-side fixed.

With the above decomposition and scaling the superalgebra (2.1) decomposes into the dynamical and kinematical superalgebras with the following anticommutators

$$\{q_{\alpha\beta}, q^{\dagger\rho\lambda}\} = P^+ \delta_\alpha^{\ \rho} \delta_\beta^{\ \lambda}\,, \quad \{q_{\alpha\beta}, q^{\dagger\dot{\alpha}\dot{\beta}}\} = 0\,, \quad \{q_{\dot{\alpha}\dot{\beta}}, q^{\dagger\dot{\rho}\dot{\lambda}}\} = P^+ \delta_{\dot{\alpha}}^{\ \dot{\rho}} \delta_{\dot{\beta}}^{\ \dot{\lambda}}\,, \tag{3.3}$$

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$$\{q_{\alpha\beta}, Q^{\dagger\dot\rho\lambda}\} = i(\sigma^i)_\alpha{}^{\dot\rho}\delta_\beta{}^\lambda K^i \quad , \quad \{q_{\alpha\beta}, Q^{\dagger\rho\dot\lambda}\} = i(\sigma^a)_\beta{}^{\dot\lambda}\delta_\alpha{}^\rho K^a \,,$$
$$\{q_{\dot\alpha\dot\beta}, Q^{\dagger\dot\rho\lambda}\} = i(\sigma^a)_{\dot\beta}{}^{\lambda}\delta_{\dot\alpha}{}^{\dot\rho} L^a \quad , \quad \{q_{\dot\alpha\dot\beta}, Q^{\dagger\rho\dot\lambda}\} = i(\sigma^i)_{\dot\alpha}{}^{\rho}\delta_{\dot\beta}{}^{\dot\lambda} L^i \,, \tag{3.4}$$

$$\begin{aligned}
\{Q_{\alpha\dot\beta}, Q^{\dagger\rho\dot\lambda}\} &= \delta_\alpha{}^\rho\delta_{\dot\beta}{}^{\dot\lambda}\ \mathbf{H} + \mu(i\sigma^{ij})_\alpha{}^\rho\delta_{\dot\beta}{}^{\dot\lambda}\ J^{ij} + \mu(i\sigma^{ab})_{\dot\beta}{}^{\dot\lambda}\delta_\alpha{}^\rho J^{ab}\,,\\
\{Q_{\alpha\dot\beta}, Q^{\dagger\dot\rho\lambda}\} &= 0\,,\\
\{Q_{\dot\alpha\beta}, Q^{\dagger\dot\rho\lambda}\} &= \delta_{\dot\alpha}{}^{\dot\rho}\delta_\beta{}^\lambda\ \mathbf{H} + \mu(i\sigma^{ij})_{\dot\alpha}{}^{\dot\rho}\delta_\beta{}^\lambda\ J^{ij} + \mu(i\sigma^{ab})_\beta{}^\lambda\delta_{\dot\alpha}{}^{\dot\rho} J^{ab}\,.
\end{aligned} \tag{3.5}$$

As it is seen from (B.8), the dynamical supercharges $Q_{\alpha\dot\beta}, Q_{\dot\alpha\beta}$ form a subalgebra/subgroup of the original PSU$(2,2|4)$, which can be identified as PSU$(2|2)\times$PSU$(2|2)\times$U$(1)_{\mathbf{H}}$. This is a superalgebra with 16 supercharges and is in fact the super-isometry of the recently obtained and explored ten dimensional LLM [22] solutions.

After this brief review of the contraction of the unextended $AdS_5\times S^5$ superalgebra, we now study the contraction of the most extended superalgebra (2.4), (2.5). We follow a similar logic. That is, first we decompose the fermionic and bosonic generators into the irreducible representations of SO(4) $\times$ SO(4) and then scale each representation in the appropriate way. The supercharges Q should, of course, be decomposed and scaled as in (3.2). The bosonic form-field extensions should then be scaled as:

$$\mathcal{R}_{-1i5a} - \mathcal{R}_{0i6a} = R^2 c_{ia}\,, \qquad \mathcal{R}_{-1i5a} + \mathcal{R}_{0i6a} = C_{ia} \tag{3.6a}$$
$$\mathcal{R}_{-1i6a} + \mathcal{R}_{0i5a} = R^2 \hat c_{ia}\,, \qquad \mathcal{R}_{-1i6a} - \mathcal{R}_{0i5a} = \hat C_{ia} \tag{3.6b}$$
$$\mathcal{Z}_{ia} + 4\mathcal{Z}_{-10i56a} = R^2 d_{ia}\,, \qquad \mathcal{Z}_{ia} - 4\mathcal{Z}_{-10i56a} = D_{ia} \tag{3.6c}$$
$$\mathcal{Z}_{-15} - \mathcal{Z}_{06} = R^2 c\,, \qquad \mathcal{Z}_{-15} + \mathcal{Z}_{06} = C \tag{3.6d}$$
$$\mathcal{Z}_{-16} + \mathcal{Z}_{05} = R^2 \hat c\,, \qquad \mathcal{Z}_{-16} - \mathcal{Z}_{05} = \hat C \tag{3.6e}$$
$$\chi + \mathcal{R}_{-1056} = R^2 z\,, \qquad \chi - \mathcal{R}_{-1056} = Z\,, \tag{3.6f}$$

while keeping c's and C's fixed when sending R to infinity. We will discuss the scaling of other $\mathcal{R}_{abij}$ components and various components of $\mathcal{Z}^{++}_{ABC\hat\mu\hat\nu\hat\rho}$ in the next subsection.

3.1 Extensions of the dynamical superalgebra

The superalgebra (2.4), (2.5) decomposes into dynamical and kinematical parts under the Inönü-Wigner contraction. The dynamical supercharges again form a subalgebra given by the following (anti)commutation relations

$$\begin{aligned}
\{Q_{\alpha\dot\beta}, Q^{\dagger\rho\dot\lambda}\} =& \delta^\rho_\alpha\delta^{\dot\lambda}_{\dot\beta}(\mathbf{H}+Z) + \mu(i\sigma^{ij})^\rho_\alpha\delta^{\dot\lambda}_{\dot\beta}\mathbf{J}_{ij} + \mu(i\sigma^{ab})^{\dot\lambda}_{\dot\beta}\delta^\rho_\alpha\mathbf{J}_{ab}\\
&+ \mu\delta^\rho_\alpha(i\sigma^{ab})^{\dot\lambda}_{\dot\beta}\mathcal{R}_{ab} - \mu(i\sigma^{ij})^\rho_\alpha\delta^{\dot\lambda}_{\dot\beta}\mathcal{R}_{ij} - \mu(i\sigma^{ij})^\rho_\alpha(i\sigma^{ab})^{\dot\lambda}_{\dot\beta}\mathcal{R}^{+-}_{ijab}
\end{aligned} \tag{3.7a}$$
$$\{Q_{\alpha\dot\beta}, Q_{\rho\dot\lambda}\} = -\mu\delta_{\alpha\rho}\delta_{\dot\beta\dot\lambda}(C+i\hat C) - 4\mu(i\sigma^{ij})_{\alpha\rho}(i\sigma^{ab})_{\dot\beta\dot\lambda}\mathcal{M}^{+-}_{ijab} \tag{3.7b}$$

$$\begin{aligned}
\{Q_{\dot\alpha\beta}, Q^{\dagger\dot\rho\lambda}\} =& \delta^{\dot\rho}_{\dot\alpha}\delta^\lambda_\beta(\mathbf{H}-Z) + \mu(i\sigma^{ij})^{\dot\rho}_{\dot\alpha}\delta^\lambda_\beta\mathbf{J}_{ij} + \mu\delta^{\dot\rho}_{\dot\alpha}(i\sigma^{ab})^\lambda_\beta\mathbf{J}_{ab}\\
&+ \mu\delta^{\dot\rho}_{\dot\alpha}(i\sigma^{ab})^\lambda_\beta\mathcal{R}_{ab} - \mu(i\sigma^{ij})^{\dot\rho}_{\dot\alpha}\delta^\lambda_\beta\mathcal{R}_{ij} - \mu(i\sigma^{ij})^{\dot\rho}_{\dot\alpha}(i\sigma^{ab})^\lambda_\beta\mathcal{R}^{-+}_{ijab}
\end{aligned} \tag{3.8a}$$

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$$\{Q_{\dot\alpha\beta}, Q_{\dot\rho\lambda}\} = -\,\mu\delta_{\dot\alpha\dot\rho}\delta_{\beta\lambda}(C - i\hat{C}) + 4\mu(i\sigma^{ij})_{\dot\alpha\dot\rho}(i\sigma^{ab})_{\beta\lambda}\mathcal{M}^{-+}_{ijab} \tag{3.8b}$$

$$\{Q_{\dot\alpha\beta}, Q^{\dagger\rho\dot\lambda}\} = \mu(\sigma^i)^{\rho}_{\dot\alpha}(\sigma^a)^{\dot\lambda}_{\beta}(C_{ia} - i\hat{C}_{ia}) \tag{3.9a}$$

$$\{Q_{\dot\alpha\beta}, Q_{\rho\dot\lambda}\} = \mu(\sigma^i)_{\dot\alpha\rho}(\sigma^a)_{\beta\dot\lambda}D_{ia} \tag{3.9b}$$

$$\{Q_{\alpha\dot\beta}, Q^{\dagger\dot\rho\lambda}\} = \mu(\sigma^i)^{\dot\rho}_{\alpha}(\sigma^a)^{\lambda}_{\dot\beta}(C_{ia} + i\hat{C}_{ia}) \tag{3.10a}$$

$$\{Q_{\alpha\dot\beta}, Q_{\dot\rho\lambda}\} = \mu(\sigma^i)_{\alpha\dot\rho}(\sigma^a)_{\dot\beta\lambda}D_{ia} \tag{3.10b}$$

In the above $\mathcal{R}_{ab} = \mathcal{R}_{-10ab}$, $\mathcal{R}_{ij} = \mathcal{R}_{ij56}$ and

$$\mathcal{R}^{s_1s_2}_{ijab} \equiv \left(\delta_{ik}\delta_{jl} + \frac{s_1}{2}\,\epsilon_{ijkl}\right)\left(\delta_{ac}\delta_{bd} + \frac{s_2}{2}\,\epsilon_{abcd}\right)\mathcal{R}_{klcd} \tag{3.11}$$

where s_1, s_2 take $\pm$ values. As we see the components of $\mathcal{R}^{s_1s_2}_{ijab}$ with $s_1s_2 = -1$ appear in the dynamical superalgebra.

The $\mathcal{M}_{ijab}$ which has appeared in (3.7b) and (3.8b) results from the $\mathcal{Z}^{++}_{\hat\mu\hat\nu\hat\rho ABC}$ extension by setting

$$\mathcal{M}_{ijab} = \mathcal{Z}^{++}_{-1ij5ab}\;. \tag{3.12}$$

Recalling the six dimensional self-duality condition (2.6), these are the only independent components of $\mathcal{Z}^{++}$ with four $so(4)\times so(4)$ indices. In the $so(4)\times so(4)$ notation $\mathcal{M}_{ijab}$ are not irreducible and one can still reduce $\mathcal{M}_{ijab}$ to self-dual and anti-self dual parts:

$$\mathcal{M}^{s_1s_2}_{ijab} \equiv \left(\delta_{ik}\delta_{jl} + \frac{s_1}{2}\,\epsilon_{ijkl}\right)\left(\delta_{ac}\delta_{bd} + \frac{s_2}{2}\,\epsilon_{abcd}\right)\mathcal{M}_{klcd}\;. \tag{3.13}$$

where the $s_1s_2 = -1$ combinations appear in the dynamical part of the superalgebra.

As we can see the (3.7a,b) and (3.8a,b) are the maximal possible extensions of the $psu(2|2)$ algebras. One should, however, note that as a result of the maximal extension the dynamical part of the extended plane-wave superalgebra is *not* a direct product of the two extended $psu(2|2)$ factors and they mix through (3.9), (3.10).

We would also like to stress that the extensions C, $\hat{C}$, and $\mathcal{M}_{ijab}$, similarly to their counterparts in the extended $psu(2|2,4)$ algebra $\mathcal{Z}_{A\hat\mu}$ and $\mathcal{Z}^{++}_{\hat\mu\hat\nu\hat\rho ABC}$, are complex valued.

3.2 Extensions of the kinematical superalgebra

Having used the above decomposition and scalings, we obtain the extended kinematical part of the plane-wave superalgebra

$$\{q_{\alpha\beta}, q^{\dagger\rho\lambda}\} = 2\delta^{\rho}_{\alpha}\delta^{\lambda}_{\beta}(P^+ + z) + (i\sigma^{ij})^{\rho}_{\alpha}\;(i\sigma^{ab})^{\lambda}_{\beta}\;r^{++}_{ijab} \tag{3.14a}$$

$$\{q_{\alpha\beta}, q_{\rho\lambda}\} = \frac{-1}{\mu}\delta_{\alpha\rho}\delta_{\beta\lambda}(c - i\hat{c}) + (i\sigma^{ij})_{\alpha\rho}\;(i\sigma^{ab})_{\beta\lambda}\;m^{++}_{ijab} \tag{3.14b}$$

$$\{q_{\dot\alpha\dot\beta}, q^{\dagger\dot\rho\dot\lambda}\} = 2\delta^{\dot\rho}_{\dot\alpha}\delta^{\dot\lambda}_{\dot\beta}(P^+ - z) - (i\sigma^{ij})^{\dot\rho}_{\dot\alpha}\;(i\sigma^{ab})^{\dot\lambda}_{\dot\beta}\;r^{--}_{ijab} \tag{3.15a}$$

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$$\{q_{\dot\alpha\dot\beta}, q_{\dot\rho\dot\lambda}\} = \frac{-1}{\mu}\delta_{\dot\alpha\dot\rho}\delta_{\dot\beta\dot\lambda}(c+i\hat{c}) + (i\sigma^{ij})_{\dot\alpha\dot\rho}\ (i\sigma^{ab})_{\dot\beta\dot\lambda}\ m^{--}_{ijab}. \tag{3.15b}$$

$$\{q_{\alpha\beta}, q^{\dagger\dot\rho\dot\lambda}\} = \frac{1}{\mu}(\sigma^i)^{\dot\rho}_{\alpha}(\sigma^a)^{\dot\lambda}_{\beta}(c_{ia} - i\hat{c}_{ia}) \tag{3.16a}$$

$$\{q_{\alpha\beta}, q_{\dot\rho\dot\lambda}\} = \mu(\sigma^i)_{\alpha\dot\rho}(\sigma^a)_{\beta\dot\lambda}d_{ia} \tag{3.16b}$$

$$\{q_{\dot\alpha\dot\beta}, q^{\dagger\rho\lambda}\} = \frac{1}{\mu}(\sigma^i)^{\rho}_{\dot\alpha}(\sigma^a)^{\lambda}_{\dot\beta}(c_{ia} + i\hat{c}_{ia}) \tag{3.17a}$$

$$\{q_{\dot\alpha\dot\beta}, q_{\rho\lambda}\} = \mu(\sigma^i)_{\dot\alpha\rho}(\sigma^a)_{\dot\beta\lambda}d_{ia} \tag{3.17b}$$

In the above r_{ijab} and m_{ijab} are obtained from $\mathcal{R}^{s_1s_2}_{ijab}$ and $\mathcal{M}^{s_1s_2}_{ijab}$ after the following rescalings:

$$r^{s_1s_2}_{ijab} = \frac{1}{\mu R^2}\mathcal{R}^{s_1s_2}_{ijab}, \quad m^{s_1s_2}_{ijab} = \frac{1}{\mu R^2}\mathcal{M}^{s_1s_2}_{ijab}, \qquad s_1s_2 = +1 \tag{3.18}$$

There are of course non-vanishing kinematical-dynamical anticommutators which we do not present here and can be worked out in a similar way.

4. Extensions from Tiny Graviton Matrix Theory

In the previous section through the Penrose limit process we obtained the plane-wave superalgebra and its most general extension. In this section we try to realize this superalgebra as the symmetry of a physical theory. Obviously this physical theory should be related to string theory (and possibly to its extended objects) on the plane-wave background. A realization of the unextended plane-wave supersymmetry algebra (3.3)-(3.5) algerba in terms of the worldsheet coordinates of the strings has been the guiding principle in obtaining the light-cone plane-wave string field theory (see section 8 of [16] for a detailed review). In this formulation, being a perturbative string theory formulation, however, the extensions which correspond to extended objects (BPS branes) are absent.

In [14] a non-perturbative formulation of plane-wave string theory was proposed, according which the DLCQ of type IIB on the plane-wave background has Matrix quantum mechanics (a $0+1$ dimensional $\mathrm{U}(J)$ gauge theory) description, the tiny graviton matrix theory (TGMT). For convenience and completeness, a very short introduction to the tiny graviton Matrix theory is given in the appendix B.

In this setup, being a non-perturbative description, one would expect the extensions to appear naturally. In this section our aim is to obtain explicit expressions for some of the extensions (brane charges) in terms of the Matrix degrees of freedom. As a DLCQ description, we would primarily be interested in the extension to the dynamical part of the supersymmetry algebra. The extensions, being correlated with physical observables, should appear as gauge invariant combination of the $J\times J$ matrices of the TGMT, and as in the BFSS case [23, 24], are generically in the form of trace of commutators.[3] In the TGMT,

[3]For the DLCQ of M-theory on the eleven dimensional plane-wave there also exists a Matrix theory, usually known as BMN or plane-wave matrix model [15]. The extensions of the eleven dimensional superalgebra in the context of BMN matrix has been discussed in [25, 26].

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however, besides the usual commutators we also have the option of the four brackets (see the appendix B). Hence, these extensions, which generically correspond to brane charges, are vanishing for finite size matrices. (Recall that in Matrix theory the space integration over world-volume (of total derivatives) is replaced with the trace over U(J) indices (of commutators).)

4.1 Computation of extensions in terms of matrices

Let us start with the expression for the dynamical supercharges given in [14]:

$$
\begin{aligned}
Q_{\dot\alpha\beta} = \sqrt{\frac{R_-}{2}}\ \mathrm{Tr}\ \Bigg[& \left(\Pi^i - \frac{i\mu}{R_-}X^i\right)(\sigma^i)^{\rho}_{\dot\alpha}\theta_{\rho\beta} + \left(\Pi^a - \frac{i\mu}{R_-}X^a\right)(\sigma^a)^{\dot\rho}_{\beta}\theta_{\dot\alpha\dot\rho} \\
& - \frac{i}{3!g_s}\left(\epsilon^{ijkl}[X^i, X^j, X^k, \mathcal{L}_5](\sigma^l)^{\rho}_{\dot\alpha}\theta_{\rho\beta} + \epsilon^{abcd}[X^a, X^b, X^c, \mathcal{L}_5](\sigma^d)^{\dot\rho}_{\beta}\theta_{\dot\alpha\dot\rho}\right) \\
& + \frac{1}{2g_s}\left([X^i, X^a, X^b, \mathcal{L}_5](\sigma^i)^{\rho}_{\dot\alpha}(i\sigma^{ab})^{\gamma}_{\beta}\theta_{\rho\gamma} + [X^a, X^i, X^j, \mathcal{L}_5](\sigma^a)^{\dot\gamma}_{\beta}(i\sigma^{ij})^{\dot\rho}_{\dot\alpha}\theta_{\dot\rho\dot\gamma}\right)\Bigg]
\end{aligned} \tag{4.1}
$$

$$
\begin{aligned}
Q_{\alpha\dot\beta} = \sqrt{\frac{R_-}{2}}\ \mathrm{Tr}\ \Bigg[& \left(\Pi^i - \frac{i\mu}{R_-}X^i\right)(\sigma^i)^{\dot\rho}_{\alpha}\theta_{\dot\rho\dot\beta} + \left(\Pi^a - \frac{i\mu}{R_-}X^a\right)(\sigma^a)^{\rho}_{\dot\beta}\theta_{\alpha\rho} \\
& - \frac{i}{3!g_s}\left(\epsilon^{ijkl}[X^i, X^j, X^k, \mathcal{L}_5](\sigma^l)^{\dot\rho}_{\alpha}\theta_{\dot\rho\dot\beta} + \epsilon^{abcd}[X^a, X^b, X^c, \mathcal{L}_5](\sigma^d)^{\rho}_{\dot\beta}\theta_{\alpha\rho}\right) \\
& + \frac{1}{2g_s}\left([X^i, X^a, X^b, \mathcal{L}_5](\sigma^i)^{\dot\rho}_{\alpha}(i\sigma^{ab})^{\dot\gamma}_{\dot\beta}\theta_{\dot\rho\dot\gamma} + [X^a, X^i, X^j, \mathcal{L}_5](\sigma^a)^{\gamma}_{\dot\beta}(i\sigma^{ij})^{\rho}_{\alpha}\theta_{\rho\gamma}\right)\Bigg]
\end{aligned} \tag{4.2}
$$

In our conventions the complex conjugate † just rises the lower indices and vice versa. Note that supercharges are trace of corresponding densities

$$
Q_{\alpha\dot\beta} = \mathrm{Tr}\ (Q_{\alpha\dot\beta})^p{}_q = (Q_{\alpha\dot\beta})^p{}_p\,, \qquad Q_{\dot\alpha\beta} = \mathrm{Tr}\ (Q_{\dot\alpha\beta})^p{}_q = (Q_{\alpha\dot\beta})^p{}_p \tag{4.3}
$$

where $p, q, r, s = 1, 2, \ldots, J$ are matrix indices.

In [14], the above expressions were proposed on the basis that their anticommutator produced the correct Hamiltonian (among other operators). Moreover, they have the right behavior under the symmetries, especially under the $\mathbb{Z}_2$ symmetry which exchanges $X^i \leftrightarrow X^a$ and the $Q_{\alpha\dot\beta}$ and $Q_{\dot\alpha\beta}$. To find the explicit form of (some of) the extensions we perform a careful computation of various anticommutators of the above dynamical supercharges. (A similar calculation can be carried out for kinematical and mixed supercharges.) For the computation we use the following basic operatorial (to be compared with matrix) commutation relations:

$$
\begin{aligned}
[X^I_{pq}, \Pi^J_{rs}] &= i\delta^{IJ}\ \delta_{ps}\delta_{qr} \\
\{(\theta^{\dagger\alpha\beta})_{pq}, (\theta_{\rho\gamma})_{rs}\} &= \delta^{\alpha}_{\rho}\delta^{\beta}_{\gamma}\ \delta_{ps}\delta_{qr} \\
\{(\theta^{\dagger\dot\alpha\dot\beta})_{pq}, (\theta_{\dot\rho\dot\gamma})_{rs}\} &= \delta^{\dot\alpha}_{\dot\rho}\delta^{\dot\beta}_{\dot\gamma}\ \delta_{ps}\delta_{qr}
\end{aligned} \tag{4.4}
$$

After some straightforward, but lengthy, algebra one can check that

$$
\{Q_{\dot\alpha\beta}, Q^{\dagger\dot\rho\lambda}\} = \delta^{\dot\rho}_{\dot\alpha}\delta^{\lambda}_{\beta}\mathbf{H} + \mu(i\sigma^{ij})^{\dot\rho}_{\dot\alpha}\delta^{\lambda}_{\beta}\mathbf{J}_{ij} + \mu\delta^{\dot\rho}_{\dot\alpha}(i\sigma^{ab})^{\lambda}_{\beta}\mathbf{J}_{ab} - \mu(i\sigma^{ij})^{\dot\rho}_{\dot\alpha}(i\sigma^{ab})^{\lambda}_{\beta}\mathcal{R}_{ijab} \tag{4.5}
$$

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and similarly for $\{Q_{\alpha\dot{\beta}}, Q^{\dagger\rho\dot{\lambda}}\}$, where $\mathbf{H}$, $\mathbf{J}_{ij}$ and $\mathbf{J}_{ab}$ are given in (B.1), (B.10) and (B.11) of the appendix B. From the above form of the supercharges it is readily seen that

$$\mathcal{R}_{ijab} = \frac{1}{g_s}\mathrm{Tr}\,\left([X^i, X^j, X^a, X^b]\mathcal{L}_5\right) \tag{4.6}$$

Next, one may show that

$$\begin{gathered}\{Q_{\dot{\alpha}\beta}, Q_{\dot{\rho}\lambda}\} = 0\ , \qquad \{Q_{\alpha\dot{\beta}}, Q_{\rho\dot{\lambda}}\} = 0\,, \\ \{Q_{\dot{\alpha}\beta}, Q_{\rho\dot{\lambda}}\} = 0\ .\end{gathered} \tag{4.7}$$

from which we learn that in the TGMT defined via the Hamiltonian (B.1)

$$C = \hat{C} = 0\,, \qquad D_{ia} = 0\,, \qquad \mathcal{M}_{ijab} = 0\ . \tag{4.8}$$

And finally one can, in a similar way show that

$$\{Q_{\dot{\alpha}\beta}, Q^{\dagger\rho\dot{\lambda}}\} \equiv \mu(\sigma^i)^{\rho}_{\dot{\alpha}}(\sigma^a)^{\dot{\lambda}}_{\beta}(C_{ia} - i\hat{C}_{ia}) \tag{4.9}$$

From the above and recalling that $\hat{C}_{ia}$ and C_{ia} are both hermitian (note (2.4) and (3.6a,b)) one can read the explicit form of C_{ia} and $\hat{C}_{ia}$

$$\begin{aligned} C^{ia} &= \frac{R_-}{\mu}\mathrm{Tr}\,\Bigg[P^iP^a - \left(\frac{1}{2g_s}\right)^2 \epsilon^{abcd}\epsilon^{ijkl}[X^j, X^b, X^c, \mathcal{L}_5][X^d, X^k, X^l, \mathcal{L}_5] \qquad (4.10)\\ &\qquad \left(\frac{\mu}{R_-}X^i + \frac{1}{3!g_s}\epsilon^{ijkl}[X^j, X^k, X^l, \mathcal{L}_5]\right)\left(\frac{\mu}{R_-}X^a + \frac{1}{3!g_s}\epsilon^{abcd}[X^b, X^c, X^d, \mathcal{L}_5]\right)\Bigg] \\ \hat{C}^{ia} &= \frac{R_-}{\mu}\frac{1}{2g_s}\mathrm{Tr}\,\left(\epsilon^{ijkl}P^j[X^a, X^k, X^l, \mathcal{L}_5] + \epsilon^{abcd}P^c[X^i, X^c, X^d, \mathcal{L}_5]\right) \qquad (4.11)\end{aligned}$$

We would like to stress that in the above computations we have assumed the finite J condition and have set Tr $([A, B])$ and Tr $([A, B, C, D])$ equal to zero. Moreover, in the above computations we have explicitly used the Gauss law constraint (B.3).

4.2 Physical interpretation of the extensions

Having computed the extensions using the expression of the supercharges (4.2) and (4.1), here we discuss some of their physical aspects:

• As we can explicitly see from the above computations some of the extensions are found to be zero (for finite size matrices). Noting the equations (3.6) it is seen that all the non-vanishing extensions are coming from the $\mathcal{R}_{AB\hat{\mu}\hat{\nu}}$ extension of the original extended $psu(2, 2|4)$ algebra (*cf.* (2.4)). As we discussed in section 2 this extension corresponds to three brane charges. The extensions χ, $\mathcal{Z}_{A\hat{\mu}}$, $\mathcal{Z}^{++}_{ABC\hat{\mu}\hat{\nu}\hat{\rho}}$ are all vanishing, indicating that in the current form of the TGMT -1, 1, and 5 branes are not present.

The above was of course expected recalling that the TGMT was obtained by the quantization (discretization) of a three brane in the plane-wave background and that in the corresponding Born-Infeld action only the contribution of the RR four-form to the Wess-Zumino terms were included [14].

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• The extensions account for the three brane RR *dipole* moments, as well as charges.

In the usual superPoincaré algebras the p-form (central) extensions are identified with the RR charge of *flat* Dp-branes. One of the consequences of this identification is that flat branes in a flat Minkowski background are 1/2 BPS objects. In a non-flat background, such as $AdS_5 \times S^5$ or the ten dimensional plane-wave, the half BPS objects are not flat branes. These are spherical three branes, the giant gravitons [27]. The three brane giant gravitons, being spherical, do not carry a net RR charge of the (selfdual) RR four-form. They, however, carry dipole moment of the four-form. For the spherical branes one can compute the dipole moment corresponding to the brane. This dipole moment is then naturally a five-form. Noting that each small element on the spherical brane locally behaves as a three brane which carries a unit of the corresponding RR charge density, one concludes that the RR dipole moment should be proportional to the volume form of the embedding space. To make the above more quantitative, recall that in a three brane action the coupling of a three brane to external RR four form is of the form

$$S_{C_4} = \int d\tau d^3\sigma \epsilon_{rsp} C_{\mu\nu\rho\alpha} \partial_\tau X^\mu \partial_{\sigma_r} X^\nu \partial_{\sigma_p} X^\rho \partial_{\sigma_s} X^\alpha \,,$$

where the Greek indices run over $0, \ldots, 9$, X^μ are the embedding coordinates of the brane and r, p, s indices run over $1, 2, 3$. If we fix the light-cone gauge, assuming that our C_4 field has a constant field strength $F^{(5)}$, we arrive at [14, 28]

$$\begin{aligned} S_{C_4} &= \int d\tau d^3\sigma \epsilon_{rsp} \; F^{(5)}_{+IJKL} \; X^I \partial_{\sigma_r} X^J \partial_{\sigma_p} X^K \partial_{\sigma_s} X^L \\ &= \int d\tau d^3\sigma \; F^{(5)}_{+IJKL} \; X^I \{X^J, X^K, X^L\} \end{aligned} \tag{4.12}$$

where $I, J, K, L = 1, 2, \ldots 8$ are the transverse light-cone coordinates and the $\{\cdot, \cdot, \cdot\}$ is the Nambu 3-bracket. From the above it is readily seen that the RR dipole moment fiveform of the brane is

$$d^{(5)}_{+IJKL} = \int d^3\sigma \; X^I \{X^J, X^K, X^L\}$$

Upon the quantization prescription discussed in [14] the fiveform dipole moment in the TGMT takes the form

$$d^{(5)}_{IJKL} \mid_{TGMT} = \text{Tr }(X^I [X^J, X^K, X^L, \mathcal{L}_5]) = \text{Tr }([X^I, X^J, X^K, X^L] \mathcal{L}_5) \tag{4.13}$$

We should emphasize that the above dipole moment expression should only be used for the transverse branes, those which contain the (light-cone) time direction X^+, but not the X^-.

The above arguments is not limited to three branes and can be repeated for any kind of D-brane alike. For the *even* dimensional branes this has been previously discussed in the literature and known as the Myers dielectric effect [29].

Before moving to some specific examples we would like to briefly discuss the matrix $\mathcal{L}_5$ which has appeared in the dipole moment expression. As noted in [14] in order to pass from the classical Nambu 3-brackets (and in general Nambu odd-brackets) to their quantum version we need to introduce an appropriate operator (or matrix). For our purposes this

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fixed matrix was called $\mathcal{L}_5$. In [30] a more precise definition of the $\mathcal{L}_5$ was given and its physical meaning was uncovered: $\mathcal{L}_5$ is the reminiscent of the 11^{th} circle. From the charge analysis appeared above and as can be seen from the second equality in (4.13), $\mathcal{L}_5$ is necessary to obtain a non-zero *dipole* moment. In other words, as discussed in [30], by definition $\mathcal{L}_5$ is a traceless $J \times J$ matrix which squares to one. Hence, in the diagonal basis it is has $J/2$ plus one and $J/2$ minus one eigenvalues and intuitively one may think of the positive (negative) eigenvalues corresponding to upper (lower) half of the three sphere. These two semi-spheres have equal but opposite RR charges while their contribution to the RR dipole moment is summed up. This fact is exactly reflected in the form of $\mathcal{L}_5$.

Depending on X^I to take value X^i or X^a the above dipole moment can be decomposed into various irreducible representations of $so(4) \times so(4)$. If all the X's appearing in $d^{(5)}$ are of the form of X^i or X^a, then the dipole moment $d^{(5)}$ is singlet of both of the $so(4)$'s (note that ϵ_{ijkl} or ϵ_{abcd} are singlets of both $so(4)$'s). In this case, for the 1/2 BPS spherical branes discussed in detail in [30], one can explicitly compute the dipole moment. For the 1/2 BPS spherical solutions

$$[X^i, X^j, X^k, \mathcal{L}_5] = -\frac{\mu g_s}{R_-}\epsilon_{ijkl}X^l,$$

and hence

$$\begin{aligned} d^{(5)}_{ijkl}\mid_{1/2\ BPS} &= \epsilon_{ijkl}\ \frac{\mu g_s}{R_-}\mathrm{Tr}\ (X^m X_m) \\ &= \epsilon_{ijkl}\ \left(\frac{\mu g_s}{R_-}\right)^2 \sum_{i=1}^{k} J_i^2\ . \end{aligned} \tag{4.14}$$

In the above we have considered a generic concentric configuration of k giant gravitons of radius J_i, where $\sum J_i = J$ [30]. For a single giant solution, the $X = J$ vacuum, where $k = 1$

$$d^{(5)}_{X=J} = \epsilon_{ijkl}\ \left(\frac{\mu J}{R_-} g_s\right)^2 = \epsilon_{ijkl} R^4_{giant}\,,$$

where $R^2_{giant} = \mu p^+ g_s$ with $p^+ = J/R_-$, is the radius of the giant graviton in the string units. This value of $d^{(5)}$ is the maximum value (4.14) can take. For the $X = 0$ vacuum, where $k = J$ or $J_i = 1$, we have the minimum possible dipole moment which is equal to

$$d^{(5)}_{X=0} = \epsilon_{ijkl}\ \left(\frac{\mu g_s}{R_-}\right)^2 J = \epsilon_{ijkl}\ l^4_{tiny} J\ .$$

where l_{tiny} is the size of the tiny three brane gravitons [14, 30]. The above is physically expected noting that it is the dipole moment of a single brane times their number J. The result that the dipole moments of branes are additive is generic and can be seen from (4.14), recalling that the size of a giant which carries J_i units of the light-cone momentum is $R_i^2 = \mu J_i g_s / R_-$.

It is notable that the dipole moment energy for these configurations is canceled out with the spherical brane tension [30] such that these spherical solutions are all eigenstates of the light-cone Hamiltonian with eigenvalue zero.

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Among the components of $d^{(5)}_{IJKL}$ which mix X^i and X^a directions only those with two X^i and two X^a appear as the extensions of the superalgebra in the form of $\mathcal{R}_{ijab}$. These are the RR dipole moment of the (topologically spherical) three branes embedded in both X^i and X^a direction. A detailed analysis of these states will be given in [31].

Finally we have the C_{ia} and $\hat{C}_{ia}$ extensions which correspond to *longitudinal* flat three branes containing X^i and X^a as well as X^+ and X^- directions. This can be seen tracing back the origin of the C_{ia} extension to before the Penrose limit and that C_{ia} is related to $\mathcal{R}_{0i5a}$ component of the $\mathcal{R}$ extension. (Note that x_{-1}, x_0 and x_5, x_6 combine to give x^+, x^-. The two other directions are basically transverse to the $AdS_5 \times S^5$ and do not appear at all.)

5. Discussion and outlook

In this work we have considered the most general extension of the ten dimensional plane-wave superalgebra. To obtain that we started with the corresponding superalgebra, the maximally extended $psu(2,2|4)$ algebra, and took the Penrose limit over that. Under this procedure the algebra naturally decomposes into the kinematical and dynamical parts. The dynamical part is of the form of the maximally extended $psu(2|2) \times psu(2|2) \times u(1)_{\mathbf{H}}$, as one would have expected if we started directly from the plane-wave algebra and studied its maximal extension. In other words, extending the algebra commutes with process of taking the Penrose limit. In this way, however, we have the virtue of a direct relation of the extension of the plane-wave to branes and their charges and whether these branes are longitudinal or transverse. We have shown that only the extensions corresponding to three branes appear in the dynamical part of the superalgebra appear in the present form of the TGMT; these three branes can however be transverse or longitudinal.

Given the form structure of the extensions and their $so(4) \times so(4)$ representation it is straightforward to see that the BPS branes allowed through our extended plane-wave superalgebra are in one-to-one correspondence with the BPS branes expected from string theory discussion of [32].

We gave an explicit realization of the supercharges, and in particular the dynamical ones, in the context of the Tiny Graviton Matrix Theory (TGMT). We discussed that the TGMT on the pure plane-wave background naturally admits and contains the central extension corresponding to spherical transverse three branes or flat longitudinal branes. An interesting outcome of our Matrix theory analysis is that the extensions can be related to the RR *dipole* moment of the branes as well as the usually discussed charges. In fact if the BPS brane configuration does not carry a net RR charge, as is the case with the giant gravitons in the AdS or plane-wave background, then the lowest moment, which in this case is the dipole moment, can appear as the extension in the algebra. We made this fact manifest in the TGMT. The fact that the dipole moments can also appear in the superalgebras as the extensions can have a profound effect in counting of the microstates of BPS blackholes in non-flat background. In particular it implies that there could possibly be some modifications to the no hair theorem, especially in the context of higher dimensional blackholes and blackholes in non-flat backgrounds. The possibility of appearance of the

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dipole moments in the blackhole thermodynamics has been recently discussed [33]. In light of the above discussions one may then try to account for the dipole moment effects in the blackhole thermodynamics through the superalgebra and its representations and classification of the dipole-charged BPS objects.

One of the interesting points which appears in studying the extensions of superalgebras are the concept of "fermionic brane charges" which was first noted in [5] and further discussed in [13]. It would be nice to study these fermionic brane charges further in the context of the TGMT or the BMN Matrix model [15]. These fermionic charges should show up when we compute commutator of the extensions with the supercharges. In the TGMT where we have given explicit form of both of these in terms of $J \times J$ matrices it is then straightforward to carry out such computations.

Having obtained the explicit form of the extended superalgebra in terms of the TGMT one can now extend the work of [30] to less supersymmetric configurations [31]. With the formulation at hand and with finite size matrices one can only analyze compact three brane configurations. For the infinite extent branes we should take the infinite J limit. In order to include other branes, e.g. D-strings or D5/NS5-branes we need to extend the TGMT by adding the appropriate terms which account for these objects. This is very similar to what is done for the BFSS matrix model [34].

In [30], based on the results in the 1/2 BPS sector, we proposed a triality between the TGMT, type IIB string theory and the $\mathcal{N} = 4$ SYM gauge theory. The latter two are of course related via the usual AdS/CFT. In order to provide further support for the proposal we need to find the explicit representation and manifestation of the fully extended $psu(2,2|4)$ superalgebra in terms of the (gauge invariant) operators of the $\mathcal{N} = 4$ four dimensional gauge theory. Moreover, from the TGMT side we need to have a classification of the less BPS configurations of the TGMT to match it against similar configurations in the gravity or dual gauge theory side.

A. Fermion notations

The supersymmetry algebra of $AdS_5 \times S^5$, in the context of Kac-Nahm classification of Lie superalgebras of classical type, is PSU$(2,2|4)$, which is also superalgebra of $D = 4, \mathcal{N} = 4$ superconformal field theory. This superalgebra may be represented using 4,10 or 10+2 dimensional notations according to taste. We shall present this superalgebra in the $so(4,2) \oplus so(6)$ and coset $so(4,1) \oplus so(5)$ basis.

A.1 Twelve dimensional fermions in SO$(4,2) \times$ SO(6) notations

In order to make the isometry of $AdS_5 \times S^5$ manifest, it is convenient to employ (10+2)-dimensional notation. 12-dimensional gamma matrices $\mathcal{G}^{\hat{m}}$ satisfy

$$\{\mathcal{G}^{\hat{m}}, \mathcal{G}^{\hat{n}}\} = 2g^{\hat{m}\hat{n}} \tag{A.1}$$

with $g^{\hat{m}\hat{n}} = diag(- - + + + + + + + + ++)$ and $\hat{m} = -1, 0, 1, \ldots, 10$. They can be represented in terms of six dimensional gamma matrices $\Gamma^{\hat{\mu}}$ and Γ^{A} with appropriate

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signature corresponding to $spin(4,2)$ and $spin(6)$ spin groups, as

$$\mathcal{G}^{\hat{\mu}} = \Gamma^{\hat{\mu}} \otimes \Gamma^7, \quad \mathcal{G}^A = \mathbf{1_8} \otimes \Gamma^A \tag{A.2}$$

where six dimensional chirality matrix is

$$\Gamma^7 = i\Gamma^{-1} \ldots \Gamma^4 = i\Gamma^5 \ldots \Gamma^{10} = \begin{pmatrix} \mathbf{1_4} & 0 \\ 0 & -\mathbf{1_4} \end{pmatrix} \tag{A.3}$$

with the following condition for six dimensional 8×8 gamma matrices

$$\{\Gamma^{\hat{\mu}}, \Gamma^{\hat{\nu}}\} = 2\eta^{\hat{\mu}\hat{\nu}}, \quad \{\Gamma^A, \Gamma^B\} = 2\delta^{AB} \tag{A.4}$$

For the sake of space we do everything here for SO(6), similar things would happen for SO$(4,2)$ just substitute 5 with -1 and 6 with 0, and the corresponding metric signature.

In six dimensions we are dealing with 8 component Dirac fermions, θ_μ. These Dirac spinors can be decomposed into two Weyl spinors θ_I and $\theta_{\dot{I}}$ where $I, \dot{I} = 1,2,3,4$ are fundamental, antifundamental indices. Γ matrices can be decomposed into $\Gamma^\pm$ and Γ^i, and with a convenient choice of basis they can be written as

$$\Gamma^+ = i\begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \ \Gamma^- = i\begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}, \ \Gamma^i = \begin{pmatrix} \gamma^i & 0 \\ 0 & -\gamma^i \end{pmatrix}, \ \Gamma^7 = \begin{pmatrix} \gamma^5 & 0 \\ 0 & -\gamma^5 \end{pmatrix} \tag{A.5}$$

Now γ^i's are 4×4 matrices satisfying $\{\gamma^i, \gamma^j\} = 2\delta^{ij}$ and act on the Weyl spinors. Now SO(6) Weyl spinors can be seen as Dirac spinors of SO(4) and in turn can be decomposed into two, 2 component Weyl spinors

$$\theta_I \to (\theta_\alpha,\ \theta_{\dot{\alpha}}) \tag{A.6}$$

where $\alpha, \dot{\alpha} = 1,2$. The $\gamma_{4\times4}$ matrices can also be reduced to 2×2 representations, $\sigma^i_{\alpha\dot{\alpha}}$ and $\bar{\sigma}^i_{\dot{\alpha}\alpha}$ in a convenient way, to act on Weyl spinors

$$(\gamma^i)_{I\dot{J}} = \begin{pmatrix} 0 & (\sigma^i)_{\alpha\dot{\beta}} \\ (\bar{\sigma}^i)_{\dot{\alpha}\beta} & 0 \end{pmatrix} \tag{A.7}$$

The SO$(4,2) \times$ SO(6) fermions carry spinorial indices of both of the groups. Because of 10-dim chirality and the fact that we choose positive sign for the self-dual five-form flux, out of four different possibilities, for the $AdS_5 \times S^5$ we only need fermions with the same SO$(4,2)$ and $S(6)$ chirality (see appendix B of [16]), i.e. $\theta_{I\hat{J}} \equiv \theta_I \otimes \theta'_{\hat{J}}$, and using the above decompositions it decomposed as $\theta_{I\hat{J}} \to (\theta_{\alpha\beta}, \theta_{\alpha\dot{\beta}}, \theta_{\dot{\alpha}\beta}, \theta_{\dot{\alpha}\dot{\beta}})$.

A.2 Ten dimensional fermions in SO(4) × SO(4) notation

In this part we present the superalgebra in coset SO$(9,1) \sim$ SO$(4,1) \oplus$ SO(5) basis. Then we go to onshell SO(8) representations, and finally we give SO(4) × SO(4) representations, appropriate to manifest isometries of plane-wave.

10-dimensional 32×32 gamma matrices $\Gamma^{\hat{a}}$ of $spin(9,1)$ which respect Clifford algebra

$$\{\Gamma^{\hat{a}}, \Gamma^{\hat{b}}\} = 2\eta^{\hat{a}\hat{b}} \tag{A.8}$$

can be decompose in terms of 4×4 gamma matrices of $spin(4,1)$ and $spin(5)$ as

$$\Gamma^{a} = \gamma^{a} \otimes \mathbf{1} \otimes \sigma_1, \quad \Gamma^{a'} = \mathbf{1} \otimes \gamma^{a'} \otimes \sigma_2 \tag{A.9}$$

Where each of them also satisfy

$$\{\gamma^{a}, \gamma^{b}\} = \eta^{ab}, \quad \{\gamma^{a'}, \gamma^{b'}\} = \delta^{a'b'} \tag{A.10}$$

In this convention, $\hat{a} = 0, 1, \ldots, 9$ and $a = 0, 1, \ldots, 4$ and $a' = 5, 6, \ldots, 9$. It may be interesting to note the 32 component 10-dimensional positive chirality spinor and negative chirality supercharges are decomposed as

$$\theta^{\hat{\alpha}} = \theta^{\alpha} \otimes \theta^{\alpha'} \otimes \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right), \quad Q^{\hat{\alpha}} = Q^{\alpha} \otimes Q^{\alpha'} \otimes \left(\begin{smallmatrix} 0 \\ -1 \end{smallmatrix}\right) \tag{A.11}$$

A convenient choice of basis for 32×32 Dirac matric would be

$$\Gamma^{+} = i \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \; \Gamma^{-} = i \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}, \; \Gamma^{\hat{i}} = \begin{pmatrix} \gamma^{\hat{i}} & 0 \\ 0 & -\gamma^{\hat{i}} \end{pmatrix}, \; \Gamma^{11} = \begin{pmatrix} \gamma^{9} & 0 \\ 0 & -\gamma^{9} \end{pmatrix} \tag{A.12}$$

where $\gamma^{\hat{i}}$ satisfy $\{\gamma^{\hat{i}}, \gamma^{\hat{j}}\} = 2\delta^{\hat{i}\hat{j}}$ where δ is the metric on the transverse space with $\hat{i} = 1, 2, \ldots, 8$. Dirac spinor in 10 dimensions, $\theta_{\hat{\alpha}}$, has 32 complex components. First we impose Majorana reality condition $\theta = \theta^{\dagger}$, so we remain with 32 real components. Then we demand Majorana spinors satisfy on-shell condition

$$\Gamma^{\pm}\theta^{\pm}_{\hat{\alpha}} = 0 \tag{A.13}$$

with the above decomposition for gamma matrices it can be easily seen that

$$\theta^{+}_{\hat{\alpha}} = \begin{pmatrix} \theta^{+}_{16} \\ 0 \end{pmatrix}, \qquad \theta^{-}_{\hat{\alpha}} = \begin{pmatrix} 0 \\ \theta^{-}_{16} \end{pmatrix} \tag{A.14}$$

$\theta^{\pm}_{16}$ can be thought of as SO(8) Majorana fermion, and $\gamma^{\hat{i}}$ as 16×16 real matrices. Furthermore we are dealing with type IIB string theory, so we demand both fermions have the same ten dimensional chirality, which is related to eight dimensional chirality as indicated above

$$\gamma^{9}\theta^{\pm}_{16} = \pm\theta^{\pm}_{16} \tag{A.15}$$

with 8-dimensional chirality matrix

$$\gamma^{9} = \begin{pmatrix} \mathbf{1_8} & 0 \\ 0 & -\mathbf{1_8} \end{pmatrix} \tag{A.16}$$

the solution is

$$\theta^{+}_{16} = \begin{pmatrix} \theta^{+}_{8} \\ 0 \end{pmatrix}, \qquad \theta^{-}_{16} = \begin{pmatrix} 0 \\ \theta^{-}_{8} \end{pmatrix} \tag{A.17}$$

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which are SO(8) Majorana-Weyl spinors, denoted by $\mathbf{8_s}$ and $\mathbf{8_c}$. Furthermore 16×16 dimensional gamma matrices can be reduced to 8×8 representations,

$$(\gamma^{\hat{i}})_{16} = \begin{pmatrix} 0 & (\gamma^{\hat{i}})_8 \\ (\bar{\gamma}^{\hat{i}})_8 & 0 \end{pmatrix} \tag{A.18}$$

We now transfer to $SO(4) \times SO(4)$ representations. To relate $SO(4) \times SO(4)$ fermions to those of SO(8), we note that $\mathbf{8_s}$ ($\mathbf{8_c}$) has positive (negative) chirality. SO(8) chirality matrix can be written as

$$\gamma^9 = \gamma^5 \times \gamma^{5'} \tag{A.19}$$

in terms of SO(4) chirality matrices

$$\gamma^5 = \gamma^{1234}, \qquad \gamma^{5'} = \gamma^{5678} \tag{A.20}$$

As mentioned above, we have two spinors of the same chirality, either two $\mathbf{8_s}$'s or two of $\mathbf{8_c}$'s. Let's choose them to be $\mathbf{8_s}$, and out of them we can write a complex $\mathbf{8_s}$. It can easily be seen that with the above decomposition for chirality matrix, for $\mathbf{8_s}$ the two SO(4) should have the same chirality while for $\mathbf{8_c}$ they should have the opposite chirality. Note also that an SO(4) spinor can be decomposed into two Weyl spinors. Explicitly

$$\mathbf{8_s} \rightarrow \theta_{\alpha\beta'};\ \theta_{\dot{\alpha}\dot{\beta}'} \tag{A.21}$$

$$\mathbf{8_c} \rightarrow \theta_{\alpha\dot{\beta}'};\ \theta_{\dot{\alpha}\beta'} \tag{A.22}$$

8×8 gamma matrices also reduce to

$$\gamma^i_{a\dot{a}} = \begin{pmatrix} 0 & (\sigma^i)_{\alpha\dot{\beta}}\delta^{\beta'}_{\alpha'} \\ (\sigma^i)^{\dot{\alpha}\beta}\delta^{\dot{\beta}'}_{\dot{\alpha}'} & 0 \end{pmatrix}, \quad \gamma^i_{\dot{a}a} = \begin{pmatrix} 0 & (\sigma^i)_{\alpha\dot{\beta}}\delta^{\dot{\beta}'}_{\dot{\alpha}'} \\ (\sigma^i)^{\dot{\alpha}\beta}\delta^{\beta'}_{\alpha'} & 0 \end{pmatrix} \tag{A.23}$$

$$\gamma^{i'}_{a\dot{a}} = \begin{pmatrix} -\delta^{\beta}_{\alpha}(\sigma^{i'})_{\alpha'\dot{\beta}'} & 0 \\ 0 & \delta^{\dot{\beta}}_{\dot{\alpha}}(\sigma^{i'})^{\dot{\alpha}'\beta'} \end{pmatrix}, \quad \gamma^{i'}_{\dot{a}a} = \begin{pmatrix} -\delta^{\beta}_{\alpha}(\sigma^{i})_{\dot{\alpha}'\beta'} & 0 \\ 0 & \delta^{\dot{\beta}}_{\dot{\alpha}}(\sigma^{i'})^{\dot{\alpha}'\beta'} \end{pmatrix} \tag{A.24}$$

with

$$(\sigma^i)_{\alpha\dot{\alpha}} = (\mathbf{1}, i\vec{\sigma})_{\alpha\dot{\alpha}}, \quad (\sigma^i)_{\dot{\alpha}\alpha} = (\mathbf{1}, -i\vec{\sigma})_{\dot{\alpha}\alpha} \tag{A.25}$$

For simplicity in the notation one may also drop the primes on the second $so(4)$ indices, whereby we arrive at the fermionic notations employed in the main text of this paper.

B. Tiny Graviton Matrix Theory, a short review

In this appendix we briefly review the basics of the tiny graviton matrix theory, TGMT. It is essentially a very short summary of [14]. The tiny graviton matrix theory proposal is that the DLCQ of strings on the $AdS_5 \times S^5$ or on the 10 dimensional plane-wave background in the sector with J units of light-cone momentum is described by the theory or dynamics of J "tiny" (three-brane) gravitons. The action for J tiny gravitons is obtained as a regularized (quantized) version of D3-brane light-cone Hamiltonian, as has been carried out in [14]. In other words, DLCQ of type IIB strings on the plane-wave background is nothing but a quantized 3-brane theory. Then the statement of the conjecture is:

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The theory of J tiny three-brane gravitons, which is a $\mathrm{U}(J)$ *supersymmetric quantum mechanics with the* $\mathrm{PSU}(2|2) \times \mathrm{PSU}(2|2) \times \mathrm{U}(1)$ *symmetry, is the Matrix theory describing the DLCQ of strings on the plane-waves or on the* $AdS_5 \times S^5$ *in the sector with light-cone momentum* $p^+ = J/R_-$*,* R_- *being the light-like compactification radius.*

Dynamics of the theory is governed by the following Hamiltonian

$$\begin{aligned}
\mathbf{H} = R_-\ \mathrm{Tr}\ \Bigg[& \frac{1}{2}(P_i^2 + P_a^2) + \frac{1}{2}\left(\frac{\mu}{R_-}\right)^2 (X_i^2 + X_a^2) \\
& + \frac{1}{2\cdot 3! g_s^2}\Big([X^i, X^j, X^k, \mathcal{L}_5][X^i, X^j, X^k, \mathcal{L}_5] + [X^a, X^b, X^c, \mathcal{L}_5][X^a, X^b, X^c, \mathcal{L}_5]\Big) \\
& + \frac{1}{2\cdot 2 g_s^2}\Big([X^i, X^j, X^a, \mathcal{L}_5][X^i, X^j, X^a, \mathcal{L}_5] + [X^a, X^b, X^i, \mathcal{L}_5][X^a, X^b, X^i, \mathcal{L}_5]\Big) \\
& - \frac{\mu}{3! R_- g_s}\left(\epsilon^{ijkl} X^i [X^j, X^k, X^l, \mathcal{L}_5] + \epsilon^{abcd} X^a [X^b, X^c, X^d, \mathcal{L}_5]\right) \\
& + \left(\frac{\mu}{R_-}\right)\left(\theta^{\dagger\alpha\beta}\theta_{\alpha\beta} - \theta_{\dot\alpha\dot\beta}\theta^{\dagger\dot\alpha\dot\beta}\right) \\
& + \frac{1}{2g_s}\left(\theta^{\dagger\alpha\beta}(\sigma^{ij})_\alpha^{\ \delta}[X^i, X^j, \theta_{\delta\beta}, \mathcal{L}_5] + \theta^{\dagger\alpha\beta}(\sigma^{ab})_\alpha^{\ \delta}\,[X^a, X^b, \theta_{\delta\beta}, \mathcal{L}_5]\right) \\
& - \frac{1}{2g_s}\left(\theta_{\dot\delta\dot\beta}(\sigma^{ij})_{\dot\alpha}^{\ \dot\delta}\,[X^i, X^j, \theta^{\dagger\dot\alpha\dot\beta}, \mathcal{L}_5] + \theta_{\dot\delta\dot\beta}(\sigma^{ab})_{\dot\alpha}^{\ \dot\delta}\,[X^a, X^b, \theta^{\dagger\dot\alpha\dot\beta}, \mathcal{L}_5]\right)\Bigg]
\end{aligned} \tag{B.1}$$

The $\mathcal{L}_5$ is a hermitian $J \times J$ defined by [30]

$$\mathcal{L}_5^2 = \mathbf{1}\,, \qquad \mathrm{Tr}\ \mathcal{L}_5 = 0\ . \tag{B.2}$$

The $\mathrm{U}(J)$ gauge symmetry of the above Hamiltonian is in fact a discretized (quantized) form of the spatial diffeomorphisms of the 3-brane. As is evident from the above construction we expect in $J \to \infty$ limit to recover the diffeomorphisms. One should note that under the $\mathrm{U}(J)$ transformations $\mathcal{L}_5$ is also transformaing in the adjoint representation.

The Hamiltonian can also be obtained from a $0+1$ dimensional $\mathrm{U}(J)$ gauge theory Lagrangian, in the temporal gauge. Explicitly, the only component of the gauge field, $\mathcal{A}_0$, has been set to zero. To ensure the $\mathcal{A}_0 = 0$ gauge condition, all of our physical states must satisfy the Gauss law constraint arising from equations of motion of $\mathcal{A}_0$. These constraints, which consist of $J^2 - 1$ independent conditions are:

$$i[X^i, P^i] + i[X^a, P^a] + \{\theta^{\dagger\alpha\beta}, \theta_{\alpha\beta}\} + \{\theta^{\dagger\dot\alpha\dot\beta}, \theta_{\dot\alpha\dot\beta}\} = 0 \tag{B.3}$$

where $P^I = D_0 X^I = \partial_0 X^I + i[\mathcal{A}_0, X^I]$ and all the fields $X, P, \theta, \mathcal{A}_0$ and $\mathcal{L}_5$ are $J \times J$ matrices.

The Hamiltonian is proposed to describe type IIB string theory on the plane-wave with compact X^- direction. The "string theory limit" is then a limit where we decompactify R_-, keeping p^+ fixed, i.e.

$$J, R_- \to \infty, \qquad \mu,\ p^+ = J/R_-, g_s\ \ \text{fixed}\ . \tag{B.4}$$

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In fact one can show that in the above string theory limit one can re-scale X's such that μ, p^+ only appear in the combination μp^+. Therefore the only parameters of the continuum theory are μp^+ and g_s.

The plane-wave is a maximally supersymmetric one, i.e. it has 32 fermionic isometries which can be arranged into two sets of 16, the kinematical supercharges, q's, and the dynamical supercharges, Q's. The former are those which anticommute to light-cone momentum P^+ and the latter anticommute to the light-cone Hamiltonian $\mathbf{H}$. Here we show the dynamical part of superalgebra, which can be identified with PSU(2|2) $\times$ PSU(2|2) $\times$ U(1):

$$\begin{aligned} [P^+, q_{\alpha\beta}] &= 0 \ , \qquad & [P^+, q_{\dot\alpha\dot\beta}] &= 0 \, , \\ [\mathbf{H}, q_{\alpha\beta}] &= -i\mu q_{\alpha\beta} \ , \qquad & [\mathbf{H}, q_{\dot\alpha\dot\beta}] &= i\mu q_{\dot\alpha\dot\beta} \ . \end{aligned} \tag{B.5}$$

$$\begin{aligned} [P^+, Q_{\alpha\dot\beta}] &= 0 \ , \qquad & [P^+, Q_{\dot\alpha\beta}] &= 0 \\ [\mathbf{H}, Q_{\alpha\dot\beta}] &= 0 \ , \qquad & [\mathbf{H}, Q_{\dot\alpha\beta}] &= 0 \end{aligned} \tag{B.6}$$

$$\{q_{\alpha\beta}, q^{\dagger\rho\lambda}\} = 2P^+ \delta_\alpha^{\ \rho} \delta_\beta^{\ \lambda} \ , \quad \{q_{\alpha\beta}, q^{\dagger\dot\alpha\dot\beta}\} = 0 \, , \quad \{q_{\dot\alpha\dot\beta}, q^{\dagger\dot\rho\dot\lambda}\} = 2P^+ \delta_{\dot\alpha}^{\ \dot\rho} \delta_{\dot\beta}^{\ \dot\lambda} \ , \tag{B.7}$$

$$\begin{aligned} \{Q_{\alpha\dot\beta}, Q^{\dagger\rho\dot\lambda}\} &= 2 \ \delta_\alpha^{\ \rho} \delta_{\dot\beta}^{\ \dot\lambda} \ \mathbf{H} + \mu (i\sigma^{ij})_\alpha^{\ \rho} \delta_{\dot\beta}^{\ \dot\lambda} \ \mathbf{J}^{ij} + \mu (i\sigma^{ab})_{\dot\beta}^{\ \dot\lambda} \delta_\alpha^{\ \rho} \mathbf{J}^{ab} \, , \\ \{Q_{\alpha\dot\beta}, Q^{\dagger\dot\rho\lambda}\} &= 0 \, , \\ \{Q_{\dot\alpha\beta}, Q^{\dagger\dot\rho\lambda}\} &= 2 \ \delta_{\dot\alpha}^{\ \dot\rho} \delta_\beta^{\ \lambda} \ \mathbf{H} - \mu (i\sigma^{ij})_{\dot\alpha}^{\ \dot\rho} \delta_\beta^{\ \lambda} \ \mathbf{J}^{ij} - \mu (i\sigma^{ab})_\beta^{\ \lambda} \delta_{\dot\alpha}^{\ \dot\rho} \mathbf{J}^{ab} \ . \end{aligned} \tag{B.8}$$

The generators of the above supersymmetry algebra can be realized in terms of $J \times J$ matrices as

$$\begin{aligned} P^+ = -P_- &= \frac{1}{R_-} \mathrm{Tr} \ \mathbf{1} \ , \qquad & P^- = -P_+ &= -\mathbf{H} \\ q_{\alpha\beta} &= \frac{1}{\sqrt{R_-}} \ \mathrm{Tr} \ \theta_{\alpha\beta} \ , \qquad & q_{\dot\alpha\dot\beta} &= \frac{1}{\sqrt{R_-}} \ \mathrm{Tr} \ \theta_{\dot\alpha\dot\beta} \end{aligned} \tag{B.9}$$

the rotation generators read

$$\mathbf{J}_{ij} = \frac{1}{2} \mathrm{Tr} \ \left(X^i \Pi^j - X^j \Pi^i - 2\theta^{\dagger\alpha\beta} (i\sigma^{ij})_\alpha^\rho \theta_{\rho\beta} + 2\theta^{\dagger\dot\alpha\dot\beta} (i\sigma^{ij})_{\dot\alpha}^{\dot\rho} \theta_{\dot\rho\dot\beta} \right) \tag{B.10}$$

$$\mathbf{J}_{ab} = \frac{1}{2} \mathrm{Tr} \ \left(X^a \Pi^b - X^b \Pi^a - 2\theta^{\dagger\alpha\beta} (i\sigma^{ab})_\beta^\rho \theta_{\alpha\rho} + 2\theta^{\dagger\dot\alpha\dot\beta} (i\sigma^{ab})_{\dot\beta}^{\dot\rho} \theta_{\dot\alpha\dot\rho} \right) \tag{B.11}$$

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